The ion currents in Fig. 1 are of the order of magnitude as in the past experimental data conducted with LaB_6 using halogen gas.[1] The parameters in those experiments were not optimized, so lower currents were probably measured (assuming other deionizing effects were negligible). The theory outlined in this paper enables optimization of system parameters for a negative ion source.

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Ideal Ramjet: Optimum M_∞ for Fuel Limit and Material Limit

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THE expression for the specific impulse I_{sp} is given[1] by the expression

$$I_{sp} = \frac{a_\infty M_\infty}{g_c}\left\{\sqrt{1+\frac{\Delta T}{T_{t\infty}}}-1\right\} \tag{1}$$

where a_∞ is the freestream speed of sound, M_∞ is the freestream Mach number, $T_{t\infty}$ is the freestream stagnation temperature, and ΔT is the increase in stagnation temperature due to combustion. The expression, Eq. (1), for I_{sp} is well known. Also for a given ΔT due to combustion, it is also well known that I_{sp} obtains a maximum as a function of M_∞. Also, for the material limited case where a maximum temperature, T_{max}, is specified, it is well known that I_{sp} obtains a maximum as a function of M_∞; here $T_{max} = T_{t\infty} + \Delta T$. The purpose of this Note is to present the derivation of some simple algebraic expressions for the value of M_∞ at which I_{sp} obtains a maximum for both the fuel limited and the material limited cases.

It is believed by the author that these expressions have not been derived previously, but that M_∞ for maximum I_{sp} is normally determined numerically. Thus it would be convenient to have closed form expressions for the value of M_∞ where I_{sp} reaches a maximum.

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Material Limited Case

For the material limited ideal ramjet, Eq. (1) becomes

$$I_{sp} = \frac{a_\infty M_\infty}{g_c}\left\{\sqrt{1+\frac{T_{max}-T_{t\infty}}{T_{t\infty}}}-1\right\} \tag{2}$$

or

$$I_{sp} = \frac{a_\infty M_\infty}{g_c}\left\{\sqrt{\frac{T_{max}}{T_{t\infty}}}-1\right\} \tag{3}$$

where

$$T_{t\infty} = T_\infty\left[1+\frac{\gamma-1}{2}M_\infty^2\right] \tag{4}$$

Therefore

$$I_{sp} = \frac{a_\infty M_\infty}{g_c}\left[\left\{\frac{2T_{max}}{T_\infty[2+(\gamma-1)M_\infty^2]}\right\}^{1/2}-1\right] \tag{5}$$

To find the M_∞ value at which I_{sp} is a maximum, set $dI_{sp}/dM_\infty = 0$; therefore differentiation yields

$$\frac{[2T_{max}]^{1/2} - [T_\infty\{2+(\gamma-1)M_\infty^2\}]^{1/2}}{[T_\infty\{2+(\gamma-1)M_\infty^2\}]^{1/2}} = \left[\frac{T_\infty\{2+(\gamma-1)M_\infty^2\}}{2T_{max}}\right]^{1/2}\left[\frac{2T_{max}M_\infty^2(\gamma-1)}{T_\infty\{2+(\gamma-1)M_\infty^2\}}\right] \tag{6}$$

Next multiply both sides of Eq. (6) by the denominator on the left-hand side of Eq. (6) and by $[2T_{max}]^{1/2}$. This yields

$$-[2T_{max}T_\infty\{2+(\gamma-1)M_\infty^2\}]^{1/2} = \frac{2T_{max}M_\infty^2(\gamma-1)}{\{2+(\gamma-1)M_\infty^2\}} - 2T_{max} \tag{7}$$

Next square both sides of Eq. (7) and collect terms in powers of M_∞. This process yields

$$M_\infty^6[(\gamma-1)^3] + M_\infty^4[6(\gamma-1)^2] + M_\infty[12(\gamma-1)] + 8 = 8T_{max}/T_\infty \tag{8}$$

or

$$[M_\infty^2(\gamma-1)+2]^3 = 8T_{max}/T_\infty \tag{9}$$

therefore

$$M_\infty = \left[\frac{2}{\gamma-1}\left\{\left(\frac{T_{max}}{T_\infty}\right)^{1/3}-1\right\}\right]^{1/2} \tag{10}$$

For example, consider the case where $\gamma = 1.4$, $T_\infty = 400°R$ and $T_{max} = 5000°R$; thus yields $M_\infty = 2.57$ and $I_{sp} = 104$ lbf-s/lbm (which agrees well with the values in Ref. 1). Equation (10) shows that to obtain maximum I_{sp} at the highest possible M_∞, T_∞ should be as low as possible for a fixed T_{max}.

Fuel Limited Case

For the fuel limited ideal ramjet, ΔT is fixed and I_{sp} is given by

$$I_{sp} = \frac{a_\infty M_\infty}{g_c}\left\{\left[\frac{T_\infty\{2+(\gamma-1)M_\infty^2\}+2\Delta T}{T_\infty\{2+(\gamma-1)M_\infty^2\}}\right]^{1/2}-1\right\} \tag{11}$$

To find M_∞ at which I_{sp} is a maximum, set $dI_{sp}/dM_\infty = 0$, which yields

$$\frac{[T_\infty\{2+(\gamma-1)M_\infty^2\}+2\Delta T]^{1/2}-[T_\infty\{2+(\gamma-1)M_\infty^2\}]^{1/2}}{[T_\infty\{2+(\gamma-1)M_\infty^2\}]^{1/2}}$$

$$=\left[\frac{2T_\infty\Delta TM_\infty^2(\gamma-1)}{T_\infty^2\{2+(\gamma-1)M_\infty^2\}^2}\right]\left[\frac{T_\infty\{2+(\gamma-1)M_\infty^2\}}{T_\infty\{2+(\gamma-1)M_\infty^2\}+2\Delta T}\right]^{1/2} \tag{12}$$

Now multiply both sides of Eq. (12) by the denominator on the left side of Eq. (12); next collect all square root terms to one side and all other terms to the other side of the equation; finally, square both sides and collect terms in powers of M_∞, which yields

$$M_\infty^6(\gamma-1)^3+2M_\infty^4(\gamma-1)^2$$

$$-4M_\infty^2(\gamma-1)-8[1+\Delta T/T_\infty]=0 \tag{13}$$

Now let $z=M_\infty^2(\gamma-1)$ and $c=[1+\Delta T/T_\infty]$, then Eq. (13) becomes

$$z^3+2z^2-4z-8c=0 \tag{14}$$

Notice Eq. (14) is of standard form[2] and has (for $c>1$) only one real root (physically meaningful) given by[2]

$$z=(A+B)-2/3 \tag{15}$$

therefore

$$M_\infty=\left\{\frac{1}{\gamma-1}\left[(A+B)-\frac{2}{3}\right]\right\}^{1/2} \tag{16}$$

where

$$A,B=\left\{\frac{-b}{2}\pm\left[\frac{b^2}{4}+\frac{a^3}{27}\right]^{1/2}\right\}^{1/3} \tag{17}$$

and

$$a=\frac{-16}{3} \qquad b=\frac{1}{27}\left\{88-216\left[1+\frac{\Delta T}{T_\infty}\right]\right\} \tag{18}$$

For example, consider the case where $\gamma=1.4$, $T_\infty=400°$R, and $\Delta T=3200°$R; from Eqs. (16-18) the results are $M_\infty=3.11$ and $I_{sp}=88$ lbf-s/lbm. The result for the fuel limited case is not quite as convenient as the material limited case but still only involves simple algebra.

Summary

For the cases of material limited ideal ramjet (given T_{max}) and fuel limited ideal ramjet (given ΔT), the derivation of the freestream Mach number at which the maximum specific impulse occurs has been presented. Usually these values are determined numerically. But now via Eqs. (10) and (16) these values of M_∞ can be quickly evaluated and the dependence of M_∞ on ΔT, T_{max}, and T_∞ is given in explicit form.

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Effect of Storable Propellants on Single-Stage Earth-to-Orbit Vehicles

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Introduction

PREVIOUS dual-expander[1] rocket engine concepts have utilized liquid oxygen as a common oxidizer for two fuels. Liquid hydrogen serves as a high specific-impulse fuel and as a coolant, and a hydrocarbon fuel provides high density. Interest in reducing the operational problems associated with handling cryogenic propellants led to an engine concept with some storable propellants. In this concept, N_2O_4 is used as the common oxidizer, MMH is used as the high-density fuel, and hydrogen is retained for cooling and as the high specific-impulse fuel. In the initial full-thrust mode, MMH and hydrogen are burned in parallel, with the MMH fuel providing 70% of the thrust. In the second mode, only hydrogen is burned in the common nozzle, providing an increased nozzle area ratio. The N_2O_4 and hydrogen propellant combination, as compared to oxygen and hydrogen, provides a propellant-density increase of 38%, but reduces specific impulse by about 12%. This study evaluates the vehicle-sizing impact of using this engine concept on single-stage Earth-to-orbit concepts. Two comparisons are shown. First, the storable-propellant engine is compared to an equivalent dual-expander engine using oxygen, propane, and hydrogen propellants. Then, the propane and hydrogen dual-expander engine is compared to previous results[2] with separate propane and hydrocarbon engines.

Analysis

The vehicle characteristics were calculated in the same manner as a previous study.[2] Optimized trajectories were integrated to establish the ratio of gross mass to burnout mass. The vehicle size was iterated until the mass estimates indicated a payload mass of 13.6 Mg while satisfying the trajectory mass ratio. The ratio of the mass of the high-density fuel and the oxidizer burned with it to the total propellant mass, called the high-density propellant fraction, was optimized parametrically.

Results

Figure 1 shows the results for vehicle dry mass. For the optimum high-density propellant fraction of about 0.55, the propane vehicle has a dry mass of 89 Mg, and the MMH vehicle has a dry mass of 107 Mg (20% greater). Figure 2 shows the results for vehicle gross mass. For single-stage vehicles, gross mass is not very significant and should not usually be used for comparisons. The results for gross mass are included here because the engines being compared may be considered for other staging concepts. The gross mass of a vehicle launched from a subsonic carrier aircraft is an important characteristic. The results show that the gross mass increases from 1036 to 1600 Mg, or 54%, when storable propellants are required.

The results with propane dual-expander engines are compared to previous results with separate propane and hydrogen engines in Fig. 3. The comparison indicates that the

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